Problem 2:

**Part A**: Determine the combination of ingredients that minimizes calories but meets all nutritional requirements.

1. Formulate the problem as a linear program with an objective function and all constraints.

We will define the variables thusly:

x1 = Tomato (100g)

x2 = Lettuce (100g)

x3 = Spinach (100g)

x4 = Carrot (100g)

x5 = Sunflower Seeds (100g)

x6 = Smoked Tofu (100g)

x7 = Chickpeas (100g)

x8 = Oil (100g)

**Objective Function:**

Y = min{21x1 + 16x2 + 40x3 + 41x4 + 585x5 + 120x6 + 164x7 + 884x8}, where Y is the minimum number of calories needed to meet all the nutritional requirements for the salad.

**Constraints:**

.85x1 + 1.62x2 + 2.86x3 + .93x4 + 23.4x5 + 16x6 + 9x7 15g of protein

2g of fat .33x1 + .2x2 + .39x3 + .24x4 + 48.7x5 + 5x6 + 2.6x7 + 100x8 8g of fat

4.64x1 + 2.37x2 + 3.63x3 + 9.58x4 + 15x5 + 3x6 + 27x7 4g of carbohydrates

9x1 + 28x2 + 65x3 + 69x4 + 3.8x5 + 120x6 + 78x7 ≥ 200mg of sodium

(x1 + x2) / (x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8) ≥ 40% by mass

xi ≥ 0 for all integers 1 ≤ i ≤ 8

1. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.

LINDO was used to solve the system of equations above. The input was as follows:

MIN 21 x1 + 16 x2 + 40 x3 + 41 x4 + 585 x5 + 120 x6 + 164 x7 + 884 x8

ST

0.85 x1 + 1.62 x2 + 2.86 x3 + 0.93 x4 + 23.4 x5 + 16.0 x6 + 9.00 x7 > 15

0.33 x1 + 0.20 x2 + 0.39 x3 + 0.24 x4 + 48.7 x5 + 5.00 x6 + 2.60 x7 + 100 x8 > 2

0.33 x1 + 0.20 x2 + 0.39 x3 + 0.24 x4 + 48.7 x5 + 5.00 x6 + 2.60 x7 + 100 x8 < 8

4.64 x1 + 2.37 x2 + 3.63 x3 + 9.58 x4 + 15.0 x5 + 3.00 x6 + 27.0 x7 > 4

9.00 x1 + 28.0 x2 + 65.0 x3 + 69.0 x4 + 3.80 x5 + 120 x6 + 78.0 x7 < 200

0.4 x1 - 0.4 x2 - 0.4 x3 - 0.4 x4 - 0.4 x5 - 0.4 x6 - 0.4 x7 - 0.4 x8 + x2 + x3 > 0

x1 > 0

x2 > 0

x3 > 0

x4 > 0

x5 > 0

x6 > 0

x7 > 0

x8 > 0

END

LINDO generated an optimal solution involving the following combination of ingredients:

|  |  |  |
| --- | --- | --- |
| Ingredient | Variable | Amount Allocated (in grams) |
| Tomato | x1 | 100 x 0 = 0 |
| Lettuce | x2 | 100 x 0.58548 = 58.548 |
| Spinach | x3 | 100 x 0 = 0 |
| Carrot | x4 | 100 x 0 = 0 |
| Sunflower Seeds | x5 | 100 x 0 = 0 |
| Smoked Tofu | x6 | 100 x .87822 = 87.822 |
| Chickpeas | x7 | 100 x 0 = 0 |
| Oil | x8 | 100 x 0 = 0 |

The objective function returns a value of 114.7541. This represents the minimum number of calories needed to create a salad. The salad would consist of 57g of lettuce and 87g of smoked tofu.

1. What is the cost of the low calorie salad?

(0.57 × 0.75) + (0.88 × 2.15) ≈ $2.32

**Part B:** Veronica realizes that it is also important to minimize the cost associated with the new salad. Unfortunately some of the ingredients can be expensive. Determine the combination of ingredients that minimizes cost.

1. Formulate the problem as a linear program with an objective function and all constraints.

We will define the variables thusly:

x1 = Tomato (100g)

x2 = Lettuce (100g)

x3 = Spinach (100g)

x4 = Carrot (100g)

x5 = Sunflower Seeds (100g)

x6 = Smoked Tofu (100g)

x7 = Chickpeas (100g)

x8 = Oil (100g)

**Objective Function:**

C = min{x1 + 0.75x2 + 0.5x3 + 0.5x4 + 0.45x5 + 2.15x6 + 0.95x7 + 2x8}, where C is the minimum cost of a salad that meets all the nutritional requirements.

**Constraints:**

.85x1 + 1.62x2 + 2.86x3 + .93x4 + 23.4x5 + 16x6 + 9x7 15g of protein

2g of fat .33x1 + .2x2 + .39x3 + .24x4 + 48.7x5 + 5x6 + 2.6x7 + 100x8 8g of fat

4.64x1 + 2.37x2 + 3.63x3 + 9.58x4 + 15x5 + 3x6 + 27x7 4g of carbohydrates

9x1 + 28x2 + 65x3 + 69x4 + 3.8x5 + 120x6 + 78x7 ≥ 200mg of sodium

(x1 + x2) / (x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8) ≥ 40% by mass

xi ≥ 0 for all integers 1 ≤ i ≤ 8

1. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.

LINDO was used to solve the system of equations above. The input was as follows:

MIN x1 + 0.75x2 + 0.5x3 + 0.5x4 + 0.45x5 + 2.15x6 + 0.95x7 + 2x8

ST

0.85 x1 + 1.62 x2 + 2.86 x3 + 0.93 x4 + 23.4 x5 + 16.0 x6 + 9.00 x7 > 15

0.33 x1 + 0.20 x2 + 0.39 x3 + 0.24 x4 + 48.7 x5 + 5.00 x6 + 2.60 x7 + 100 x8 > 2

0.33 x1 + 0.20 x2 + 0.39 x3 + 0.24 x4 + 48.7 x5 + 5.00 x6 + 2.60 x7 + 100 x8 < 8

4.64 x1 + 2.37 x2 + 3.63 x3 + 9.58 x4 + 15.0 x5 + 3.00 x6 + 27.0 x7 > 4

9.00 x1 + 28.0 x2 + 65.0 x3 + 69.0 x4 + 3.80 x5 + 120 x6 + 78.0 x7 < 200

0.4 x1 - 0.4 x2 - 0.4 x3 - 0.4 x4 - 0.4 x5 - 0.4 x6 - 0.4 x7 - 0.4 x8 + x2 + x3 > 0

x1 > 0

x2 > 0

x3 > 0

x4 > 0

x5 > 0

x6 > 0

x7 > 0

x8 > 0

END

LINDO generated an optimal solution involving the following combination of ingredients:

|  |  |  |
| --- | --- | --- |
| Ingredient | Variable | Amount Allocated (in grams) |
| Tomato | x1 | 100 x 0 = 0 |
| Lettuce | x2 | 100 x 0 = 0 |
| Spinach | x3 | 100 x .832298 = 83.2298 |
| Carrot | x4 | 100 x 0 = 0 |
| Sunflower Seeds | x5 | 100 x .096083 = 9.6083 |
| Smoked Tofu | x6 | 100 x 0 = 0 |
| Chickpeas | x7 | 100 x 1.152364 = 115.2364 |
| Oil | x8 | 100 x 0 = 0 |

The objective function returns a value of 1.554133 which is ≈ $1.55. This represents the minimum cost of a salad that meets the nutritional requirements. It contains 83 grams of lettuce, 9.6 grams of sunflower seeds and 115 grams of chickpeas.

1. How many calories are in the low cost salad?

(0.832298 × 40) + (0.096083 × 585) + (1.152364 × 164) ≈ 278 calories.

**Part C**: Compare the results from part A and B. Veronica’s goal is to create a Very Veggie Salad that is both low calorie and low cost. She would like to sell the salad for $5.00 and still have a profit of at least $3.00. However if she can advertise that the salad has under 250 calories then she may be able to sell more.

1. Suggest some possible ways that she select a combination of ingredients that is “near optimal” for both objectives. This is a type of multi-objective optimization.
2. Determine the minimum number of ingredients by gram that the salad can contain while meeting all nutritional requirements, with a cost no greater than $2.00 and a calorie count of no more than 250.
3. Minimize the cost of the salad while ensuring that it meets all the minimum nutritional requirements with the additional constraint that it is no more than 250 calories.
4. Find the minimum number of calories in a salad that meets all the nutritional requirements and costs no more than $2.00 to produce.
5. What combination of ingredient would you suggest and what is the associated cost and calorie.

I would suggest using approach #2 since it will derive the salad which meets our calorie requirement yet has the lowest production cost.

LINDO was used to solve problem above. The input was as follows:

MIN v1 + 0.75 v2 + 0.5 v3 + 0.5 v4 + 0.45 v5 + 2.15 v6 + 0.95 v7 + 2 v8

ST

21 v1 + 16 v2 + 40 v3 + 41 v4 + 585 v5 + 120 v6 + 164 v7 + 884 v8 < 250

0.85 v1 + 1.62 v2 + 2.86 v3 + 0.93 v4 + 23.4 v5 + 16.0 v6 + 9.00 v7 > 15

0.33 v1 + 0.20 v2 + 0.39 v3 + 0.24 v4 + 48.7 v5 + 5.00 v6 + 2.60 v7 + 100 v8 > 2

0.33 v1 + 0.20 v2 + 0.39 v3 + 0.24 v4 + 48.7 v5 + 5.00 v6 + 2.60 v7 + 100 v8 < 8

4.64 v1 + 2.37 v2 + 3.63 v3 + 9.58 v4 + 15.0 v5 + 3.00 v6 + 27.0 v7 > 4

9.00 v1 + 28.0 v2 + 65.0 v3 + 69.0 v4 + 3.80 v5 + 120 v6 + 78.0 v7 < 200

0.4 v1 - 0.4 v2 - 0.4 v3 - 0.4 v4 - 0.4 v5 - 0.4 v6 - 0.4 v7 - 0.4 v8 + v2 + v3 > 0

v1 > 0

v2 > 0

v3 > 0

v4 > 0

v5 > 0

v6 > 0

v7 > 0

v8 > 0

END

LINDO generated an optimal solution involving the following combination of ingredients:

|  |  |  |
| --- | --- | --- |
| Ingredient | Variable | Amount Allocated (in grams) |
| Tomato | x1 | 100 x 0 = 0 |
| Lettuce | x2 | 100 x 0 = 0 |
| Spinach | x3 | 100 x .761996 = 76.1996 |
| Carrot | x4 | 100 x 0 = 0 |
| Sunflower Seeds | x5 | 100 x .09383 = 9.383 |
| Smoked Tofu | x6 | 100 x .168941 = 16.8941 |
| Chickpeas | x7 | 100 x .880222 = 88.0222 |
| Oil | x8 | 100 x 0 = 0 |

Rounding to the nearest whole ingredient, we see that Veronica’s preferred salad recipe will consist of 76g of spinach, 9g of sunflower seeds, 17g of smoked tofu, and 88 grams of chickpeas.

**Calories**:

(0.76 × 40) + (0.09 × 585) + (0.17 × 120) + (0.88 × 164) = 247.77 calories

**Production Cost:**

(0.76 × 0.5) + (0.09 × 0.45) + (0.17 × 2.15) + (0.88 × 0.95) = $1.62

Veronica will make a profit of $3.38 for every salad sold and she will maximize her volume because the salad contains less than 250 calories.

1. Note: There is not one “right” answer. Discuss how you derived your solution.

The solution was derived by minimizing each suggested approach according to whichever constraints were presented. The results were then compared according to calories require and profit.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Approach | Ingredients | Calories | Cost | Profit |
| 1 | 53g spinach, 9g sunflower seeds, 72g  smoked tofu | 160.52 | $1.85 | $3.15 |
| 2 | 76g spinach, 9g sunflower seeds, 17g  smoked tofu, 88g chickpeas | 247.77 | $1.62 | $3.38 |
| 3 | 55g spinach, 3g sunflower seeds, 80g  smoked tofu | 135.55 | $2.00 | $3.00 |